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Distributed weighted fusion estimation for uncertain networked systems with transmission time-delay and cross-correlated noises[☆]

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Abstract

This paper investigates the state estimation issue for uncertain networked systems considering data transmission time-delay and cross-correlated noises. A distributed robust Kalman filtering-based perception and centralized fusion method is proposed to improve the estimation accuracy from perturbed measurement; consequently, reduce the amount of redundant information and alleviate the estimation burden. To describe the transmission time-delay and give rise to cross-correlated and state-dependent noises in the exchange measurement among neighbors, a weighted fusion reorganized innovation strategy is proposed to reduce the computational burden and suppress noise effect. Moreover, to obtain the optimal linear estimate, a fusion estimation approach is used for information collaboration by weighting the error cross-covariance matrices. Finally, an illustrative example is presented to demonstrate the effectiveness and robustness of the proposed method.

Keywords: Distributed fusion estimation, robust Kalman filtering, uncertain

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1. Introduction

The networked systems focus on large-scale information and communication technologies, for the growing information computation and collaborative perception requirement. Introducing the distributed strategy by exchanging information improves the collaborative link between physical and computational elements, as well as increases adaptability, efficiency, and autonomy [1, 2] of the systems. The distributed networked systems are widely applied in the areas of cyber-physical systems (CPSs) [1], communication networks [3], smart grids [4] and so on. As far as the networked systems, information perception effectively utilizes the interested information for estimating a quantity to achieve a satisfactory performance. Control and optimization over networked systems are often distributed in nature; therefore, the research on distributed estimation for networked systems is challenging.

The resource-constrained networked systems are influenced by noise disturbances, which are generated by the communication facilities and geographical location of sensor nodes. When the parameter uncertainties exist in the systems, the estimation performance would be deteriorated. In order to overcome the sensitivity, various noises sequences are introduced. A modified fuzzy Kalman-type filtering [5] is presented with finite-step auto-correlated process noises depending on the system state. For the linear discrete time-varying systems with stochastic uncertainties, a realistic mathematical model named the uncertain state-dependent (also called multiplicative) noise [6, 7, 8, 9] is widely applied in a range of scientific and industrial applications. The information fusion steady-state Kalman filtering approach [10] involves different local dynamic models and correlated noises. Song et al. presents Kalman filtering fusion [11] with feedback and cross-correlated sensor noises for distributed recursive state estimators. The cross-correlation between the measurement noise and process noise are discussed in [12, 13, 14]. For noise sequences with uncertain variances, the

actual filtering error variances [15, 16, 17] are obtained with a minimal upper
30 bound for all admissible uncertainties. However, the generation mechanism of
multi-step cross-correlation for noises is not clearly analyzed.

Owing to the limited communication capacity with bandwidth, network-
induced time-delays exist inevitably during data transmission; consequently,
the data received by the estimator may be asynchronous. Therefore, the de-
35 sign of the filtering algorithms to reduce the negative influences of the time-
delays has received increasing research attention. The augmented state ap-
proach [18, 19, 20] applies the partial differential equation (PDE) and boundary
condition equation, and the polynomial approach [21, 22] is utilized to solve
the multiple time-delay systems. In order to reduce the communication burden,
40 the measurement transformation approach [17, 23, 24, 25] uses the reorganized
measurement sequence, and the delayed system is transformed into the form
of the equivalent delay-free counterpart. However, the aforementioned litera-
ture is confined to the state augmentation method to deal with transmission
time-delay, and the approach increases the processing capability of estimator
45 over distributed networked systems with cross-correlated noises and transmis-
sion time-delay.

The distributed networked systems are now widely applied in a range of
areas including battlefield surveillance, target tracking and localization, envi-
ronment monitoring, fault diagnosis and so on [26, 27, 28]. Since employing the
50 distributed estimation strategy is a means of achieving collaboration and infor-
mation fusion, it increases information reliability, improves estimation accuracy,
and suppresses communication burden. To probe the effective estimation, vari-
ous distributed fusion estimation approaches for filtering have been researched
such as distributed H_2/H_∞ filtering [29, 30, 31], distributed particle filtering
55 [32], distributed Kalman filtering [13, 26, 27, 33], etc. One of the important
issues of distributed Kalman filtering is concerned with data perception and
information fusion. For the state estimation of large-scale systems, a fused dis-
tributed Kalman filter for sensor networks [26, 34] is presented to study the
convergence of the error estimation process. Jiang et al. [35] summarize some

60 Kalman filtering approaches which are mutually equivalent in a unified frame-
work. Carli et al. [36] propose a two stage estimation method: a Kalman-like
non-communication measurement update and an estimate fusion using a con-
sensus matrix, which is designed by optimizing the steady state prediction (or
estimation) error. The distributed averaging consensus over a network [37] is
65 represented to achieve the fast converge linear iteration. Dual et al. [38] inves-
tigate lossless linear transformation for distributed estimation fusion to solve
the case of a singular covariance matrix of measurement noise. A quantiser for
communication is embedded in each sensor network to produce local estimates
using Kalman filtering scheme [33].

70 Based on the aforementioned literature, there is a great number of challeng-
ing issues for correlated noises and network-induced transmission time-delay.
However, the data fusion estimation problem for the distributed uncertain net-
worked systems with compensation multi-step random delays, stochastic uncer-
tainties, auto-correlated and cross-correlated noises has not been considered yet.
75 The compensation for multi-step random delays model is a challenging scheme;
in particular, the cross-correlated noises together with state-dependent noise are
rarely mentioned. Motivated by the above discussion, this paper is concerned
with modeling and probing the distributed fusion analysis method. Obtained
result is proved theoretically to be more accurate estimation performance than
80 some existing results, such as the weighted robust Kalman filter [13], optimal se-
quential fusion method [14], and so on. In addition, the distributed data fusion
estimation approach can be applied to radar tracking, communication system,
fault diagnosis and so on. The main contributions of this paper are summarized
as follows:

- 85 • A stochastic model is established to describe a class of distributed net-
worked systems with communication time-delay, stochastic uncertainties
and cross-correlated noises, which investigates the compensation strate-
gy by reconstructing a measurement delayed system with state-dependent
and cross-correlated noises to alleviate computation burden.

- 90 • The information exchanges between neighboring sensor nodes, and a novel weighted fusion reorganized innovation sequence is used to combine cross-correlated noises by the one-step prediction-based compensation scheme. The fused innovation sequence is able to suppress the negative effect of noises.
- 95 • Based on the optimal information fusion criterion, a weighted fusion filtering error cross-covariance method is presented. This method fuses the local state estimation and measurement data to acquire higher estimation accuracy than each local one.

The remainder of the paper is organized as follows. Section 2 describes
 100 the model of distributed networked systems. Section 3 presents the distributed robust Kalman-type filtering approach for distributed fusion estimation. Simulation results and analysis are given in Section 4 and the conclusions are outlined in Section 5.

2. Problem formulation

105 2.1. Distributed networked systems fusion structure

The issue of the distributed fusion structure for a class of networked systems is illustrated in Fig.1, where each sensor transmits measurement data to an information processor (i.e. estimator), and transmission time-delay inevitably exists in the communication network. Note that the considered system state
 110 can be measured by n time-synchronized sensors.

The uncertainty for the distributed stochastic system can be described by the stochastic state-dependent noise, which makes the system nonlinear. Therefore, the objective is to probe an approximate linear expression to fit the stochastic uncertainties. The considered system with state-dependent uncertainty, cross-correlated noises and transmission time-delay is described as:

$$x_{k+1} = (A_k + \Delta A_k) x_k + B_k \omega_k, \quad k = 1, 2, \dots \quad (1)$$

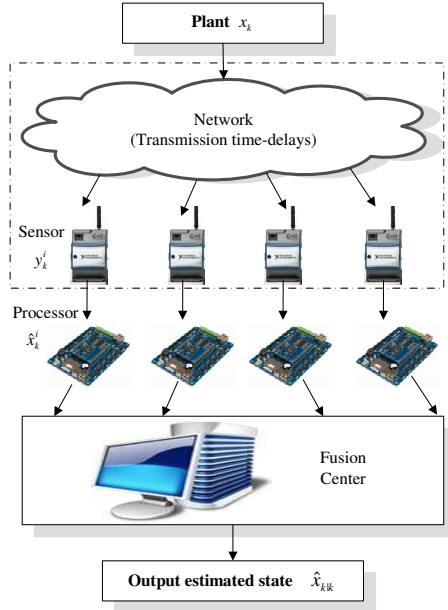


Figure 1: A distributed system structure.

$$y_k^i = H_k^i x_{k-\tau_k} + v_k^i, \quad i = 1, \dots, n, \quad (2)$$

In the following, \mathbb{R}^r denotes the r -dimensional Euclidean space, $\mathbb{R}^{r \times r}$ represents the set of all $r \times r$ real matrices, and i represents the index number of a given sensor. $x_k \in \mathbb{R}^r$ is the state vector, and $y_k^i \in \mathbb{R}^{m_i}$ expresses the measurement output of the i^{th} sensor at time instant k . $A_k \in \mathbb{R}^{r \times r}$ is the state transition matrix, $B_k \in \mathbb{R}^r$ is known as the real time-varying vector, and $\omega_k \in \mathbb{R}$ expresses the process noise with covariance Q_k . The state-dependent noise $\Delta A_k = C_k \xi_k$ denotes the internal uncertainty of the system [13, 39], where $C_k \in \mathbb{R}^{r \times r}$ represents the state transition matrix, and $\xi_k \in \mathbb{R}$ is a zero-mean white noise uncorrelated with other signals.

It is assumed that the received measurement data lag behind the state updating; therefore, the measurement time-delay should be considered. From (2), $\tau_k > 0$ stands for the transmission time-delay at time instant k , $H_k^i \in \mathbb{R}^{m_i \times r}$ is

the measurement matrix, and $v_k^i \in \mathbb{R}^{m_i}$ denotes the measurement noise of the i^{th} sensor with covariance R_k^i . The initial state x_0 with mean μ_0 and covariance P_0 is assumed to be uncorrelated with other noise signals.

Introducing the distributed fusion structure given in (1) and (2), each subsystem shares its local information with all underlying neighboring sensors for gaining additional information about the system dynamics. Thus, each information processor is able to coordinate its behavior by receiving information from other sensors within a certain neighboring area. In the estimation fusion center, each sensor communicates with its neighbors to exchange information. Therefore, the distributed fusion strategy possesses the advantages in flexibility, robustness and survivability [14].

2.2. Transmission time-delay and cross-correlated noises

Typically, in a common noise environment of distributed systems, since both the process and measurement noises of each sensor are dependent on the system state, there will be cross-correlation between different sensors and cross-correlation between process noise and measurement noise [12]. Fig.2 shows the effect of the sensor network with time-delay and cross-correlated noises on the time sequence. For the distributed fusion estimation, the fusion center is used for achieving collaboration and information fusion. There are n different buffers to store the corresponding local estimation signals, and each buffer stores the most recent time-stamped data.

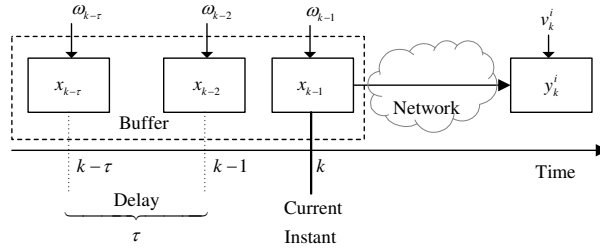


Figure 2: System with transmission delays and cross-correlated noises.

Remark 1. For the linear discrete time-varying system (1) and (2), due to the
 145 influence of transmission time-delay and the state-dependent of the system, the
 measurement is dependent on the state before being transmitted; thus, noises for
 previous sample time are more important for the current measurement. In the
 communication process, it is assumed that the process and measurement noises
 of each sensor, as well as the different sensor noises are cross-correlated at each
 150 sample time and previous sample time, i.e. the process noise at time instant k
 is correlated with the process noise at time k and $k - 1$ with the covariances
 $E(\omega_k \omega_k^T) = Q_k$ and $E(\omega_k \omega_{k-1}^T) = Q_{k-1,k}$, respectively. On the other hand,
 the transmission time-delay generates the measurement delay, the measurement
 noise at time instant k is correlated at time k and $k - 1$ with the covariances
 155 $E(v_k^i (v_k^i)^T) = R_k^i$ and $E(v_k^i (v_{k-1}^i)^T) = R_{k-1,k}^i$, respectively. Meanwhile, the
 measurement noise v_k^i and v_l^j of different sensors are cross-correlated for one
 sample time only at time instants k and $k - 1$.

Since the considered noises are cross-correlated, ω_k , v_k^i and v_l^j have the
 following statistical properties:

$$\begin{aligned}
 & E(\omega_k) = 0, \quad E(v_k^i) = 0, \\
 & E \left(\begin{bmatrix} \omega_k \\ v_k^i \end{bmatrix} \begin{bmatrix} \omega_l^T & (v_l^j)^T \end{bmatrix} \right) \\
 & = \begin{bmatrix} Q_k \delta_{k-l} + Q_{k,l} \delta_{k-l+1} & S_k^j \delta_{k-l} + S_{k,l}^j \delta_{k-l+1} \\ 0 & R_k^{i,j} \delta_{k-l} + R_{k,l}^{i,j} \delta_{k-l+1} \end{bmatrix}, \tag{3}
 \end{aligned}$$

where function $E(\cdot)$ denotes the mathematical expectation operator, the super-
 script T is the transpose, and δ_{k-l} is the Kronecker function, i.e. $\delta_{k-l} = 1$ if
 160 $k = l$, and $\delta_{k-l} = 0$ if $k \neq l$. Here, $Q_k = Q_k^T$, $R_k^i = (R_k^i)^T$ and $R_k^{i,j} = (R_k^i)^T$
 if $i = j$.

Remark 2. Taking the influence of transmission time-delay and noises cross-
 correlation into account, both the process and different sensor noises are cross-
 correlated for one sample time only, i.e. the measurement noise of the i^{th} sensor
 165 at time k is cross-correlated with the process noise at time k and $k - 1$ with the
 covariances $E(\omega_k (v_k^i)^T) = S_k^i$ and $E(\omega_k (v_{k-1}^i)^T) = S_{k-1,k}^i$, respectively. The

diagram of time sequence shows that the later-step cross-correlation for noises is redundant, i.e. the measurement noise of the i^{th} sensor at time instant k is uncorrelated with the process at time $k + 1$. Therefore, the introduced one-step cross-correlation depending on the time sequence alleviate the computational complexity.

The objective of the distributed fusion estimation is to design a robust Kalman-type filter $\hat{x}_{k|k}$ with multi-step transmission time-delay and cross-correlated noises. The appropriate filter parameter K_k^i is probed by minimizing the estimation error covariance.

3. Distributed robust Kalman-type filtering

This section investigates a distributed unbiased robust Kalman-type filtering fusion approach for minimizing the estimation error cross-covariance matrices.

3.1. Reorganized innovation sequence

The measurement output y_k^i in (2) from the i^{th} sensor is expressed by the state $x_{k-\tau_k}$ at time instant k , and the estimated value of local Kalman filter is designed as $\hat{x}_{k|k}^i$. Employing the measurement reorganization approach transforms the systems with time-delayed measurements into the equivalent systems without measurement delays. Especially when the delays are large, it is used for solving different Kalman filters with the same dimension as the original system to suppress the computational burden [17, 23]. Therefore, the system with τ_k -step time-delayed measurement is investigated from reorganized measurement output $\{y_k^i, y_{k-1}^i, \dots, y_{k-\tau_k}^i\}$ and measurement noise $\{v_k^i, v_{k-1}^i, \dots, v_{k-\tau_k}^i\}$. Note that measurement noise sequence is also white noise with zero-mean and covariance R_k^i .

3.1.1. Measurement reorganization

Given the measurement sequence $\{y_k^i, \dots, y_{k-\tau_k}^i, \dots, y_0^i\}$, the estimated value $\hat{x}_{k|k}^i$ is the projection of state x_k onto the linear spanned space as $\mathcal{L}\{y_k^i, \dots, y_{k-\tau_k}^i, \dots, y_0^i\}$. The measurement sequence with transmission

time-delay is defined as two components, i.e. $\left\{ \left\{ y_s^i \right\}_{s=0}^{k-\tau_k-1} \right\}$ and $\left\{ \left\{ y_s^i \right\}_{s=k-\tau_k}^k \right\}$. Therefore, a linear minimum mean square error (LMMSE) estimation $\hat{x}_{k|k}^i$ is defined as:

$$\hat{x}_{k|k}^i = \text{proj} \left\{ x_k | y_k^i, \dots, y_{k-\tau_k}^i, \dots, y_0^i \right\}, \quad (4)$$

which is equal to the compensation by the filtered value $\hat{x}_{k-\tau_k|k-\tau_k}^i$ (i.e. $\hat{x}_{k|k}^i = \left(\prod_{l=1}^{\tau_k} (A_{k-l}) \right) \hat{x}_{k-\tau_k|k-\tau_k}^i$) [40]. Since the measurement sequence contains τ_k -step time-delays for the state x_k , the reorganized measurement sequence is proposed to design a predictor of $\hat{x}_{k|k-\tau_k}^i$. Denote the prediction error and filtering errors as:

$$\tilde{x}_{k|k-\tau_k}^i = x_k - \hat{x}_{k|k-\tau_k}^i, \quad (5)$$

and

$$\tilde{x}_{k|k}^i = x_k - \hat{x}_{k|k}^i, \quad (6)$$

the corresponding filtering error covariance and cross-covariance matrices given in (7) and (8) are used for information exchanging between any two subsystems:

$$P_{k|k}^i = E \left(\tilde{x}_{k|k}^i \left(\tilde{x}_{k|k}^i \right)^T \right), \quad (7)$$

and

$$P_{k|k}^{i,j} = E \left(\tilde{x}_{k|k}^i \left(\tilde{x}_{k|k}^j \right)^T \right). \quad (8)$$

The aim of the distributed estimation satisfies the unbiased characteristic; meanwhile, the appropriate filter parameter K_k^i is calculated to minimize $P_{k|k}^i$.

The distributed Kalman-type filter is designed to reduce noise influence on
 195 sensor networks. In the practical engineering application, due to the cross-
 correlation for noises, the performance of traditional Kalman filter would be
 largely deteriorated, and the globally optimal Kalman filter is difficult to ob-
 200 tain. Therefore, the local robust Kalman filter with time-delay would be in-
 volved. Without loss of generality, the measurements with time-delays contain
 one-step delay (i.e. $\tau_k = 1$) and multi-step delays (i.e. $\tau_k \geq 2$). For the com-
 pensation mechanism, the distributed robust Kalman filter for linear discrete

time-varying system is transformed into the compensation by one-step prediction, which is given by the reorganized measurement sequence and reorganized innovation sequence [23, 41]. For the sake of brevity, the case of $\tau_k = 1$ will be
 205 principally investigated for the measurement delayed system.

3.1.2. Weighted fusion reorganized innovation sequence

The linear spanned space by the measurement sequence with one-step time-delay is reorganized as $\mathcal{L}\{y_k^i, \dots, y_{k-\tau_k}^i\}$. As mentioned before, the reorganized innovation sequence is expressed as $\varepsilon_k^i = y_k^i - \hat{y}_k^i$, where $\hat{y}_k^i = H_k^i \hat{x}_{k|k-1}^i$.

210 *Remark 3.* The considered distributed uncertainty system communicates with sensors; thus, each subsystem will contain the interrelated information from its communication neighbors. Assume that the networked topology is undirected graph, and two subsystems communicate with each other. In order to reduce the estimation error and improve the measurement accuracy, a robust Kalman
 215 filter is designed by the weighted fusion strategy [42] with assistance of the reorganized innovation sequence. With the aim of improving the convergence rate for the distributed fusion state estimation, a multi-innovation approach [43] is introduced. On the basis of these theories, a novel weighted fusion reorganized innovation is presented. Considering the existence of measurement time-delay
 220 and information collaboration, the proposed innovation sequence fuses noises disturbances and achieves collaboration within a certain neighboring area.

Lemma 1. For the distributed system (1) and (2), the weighted fusion reorganized innovation sequence is composed of $\{\tilde{\varepsilon}_k, \dots, \tilde{\varepsilon}_{k-\tau_k}\}$. $\tilde{\varepsilon}_k$ is a vector from $\tilde{Y}_k = [\varepsilon_k^1, \dots, \varepsilon_k^n]^T$ at time instant k , where $\varepsilon_k^i = y_k^i - \hat{y}_k^i, i = 1, \dots, n$ and $y_k^i \in \mathbb{R}^{m_i}$. Furthermore, $\tilde{\varepsilon}_k$ is represented as

$$\tilde{\varepsilon}_k = \Theta_k^T \tilde{Y}_k. \quad (9)$$

Define the weighted matrix $\Theta_k = [\theta_k^1, \dots, \theta_k^n]^T$ as

$$\Theta_k = \Sigma_k^{-1} I (I^T \Sigma_k^{-1} I)^{-1}, \quad (10)$$

where $I \triangleq \underbrace{[I_m, \dots, I_m]^T}_n$ and $\Sigma_k = E(\tilde{Y}_k \tilde{Y}_k^T)$.

Proof. For all the n subsystems, the employed sensors are homogeneous facilities with the same physical structure, i.e. $m_1 = m_2 = \dots = m_n = m$. The communication is mutual between sensor and its neighboring nodes; therefore, the fused innovation sequence $\tilde{\varepsilon}_k$ via the communication links and cross-correlated noises is represented as:

$$\tilde{\varepsilon}_k = \theta_k^1 \varepsilon_k^1 + \dots + \theta_k^n \varepsilon_k^n = \sum_{i=1}^n \theta_k^i \varepsilon_k^i = \Theta_k^T \tilde{Y}_k, \quad (11)$$

where θ_k^i , $i = 1, 2, \dots, n$ are arbitrary weighted matrices constrained by

$$\theta_k^1 + \theta_k^2 + \dots + \theta_k^n = I_m. \quad (12)$$

Let $\Theta_k = [\theta_k^1, \dots, \theta_k^n]^T$ and $I = \underbrace{[I_m, \dots, I_m]^T}_n$ be both $mn \times m$ -dimensional matrices, the covariance of the fused reorganized innovation sequence at time instant k is calculated by

$$P_k = E(\tilde{\varepsilon}_k \tilde{\varepsilon}_k^T) = \Theta_k^T \Sigma_k \Theta_k, \quad (13)$$

where $\Sigma_k = E(\tilde{Y}_k \tilde{Y}_k^T)$.

Note that $\Sigma_k = \Sigma_k^T$ is a symmetric positive definite matrix due to the noises cross-correlation, so that $I^T \Sigma_k^{-1} I$ is nonsingular. Then, referring to [42] the parameter matrix Θ_k is calculated by :

$$\Theta_k = \Sigma_k^{-1} I (I^T \Sigma_k^{-1} I)^{-1}. \quad (14)$$

The weighted fusion reorganized innovation sequence is obtained by the weighted matrix Θ_k and n innovation sequences at time instant k . This completes the derivation of the solution to Lemma 1. \square

As mentioned before, the relationship of innovation between $\tilde{\varepsilon}_k$ and ε_k^i can be directly obtained from [40], which satisfies $P_k \leq E\left(\varepsilon_k^i \left(\varepsilon_k^i\right)^T\right)$. It is implied that the proposed weighted fusion innovation sequence has superior performance to alleviate the noise disturbance. Since sensor transforming measurement to estimator communicates with its neighbors to share information, the weighted fusion innovation sequence is adopted to collaborate with its neighboring sensor nodes.

3.2. Distributed fusion estimation

235 The traditional Kalman filtering algorithm possesses good performance for unbiased estimate. Lemma 2 analyses the unbiasedness of the proposed weighted fusion method.

Lemma 2. *Considered the distributed networked system with one-step transmission time-delay, i.e. $\tau_k = 1$, for the i^{th} subsystem, the robust Kalman-type recursive filter before the fusion can be given as:*

$$\hat{x}_{k|k-1}^{i,L} = \text{proj} \{x_k | \varepsilon_{k-1}^i, \dots, \varepsilon_0^i\} = A_{k-1} \hat{x}_{k-1|k-1}^{i,L}, \quad (15)$$

$$\hat{x}_{k|k}^{i,L} = \text{proj} \{x_k | \varepsilon_k^i, \dots, \varepsilon_0^i\} = A_{k-1} \hat{x}_{k-1|k-1}^{i,L} + K_k^i \varepsilon_k^i, \quad (16)$$

where $\hat{x}_{k|k-1}^{i,L}$ and $\hat{x}_{k|k}^{i,L}$ are the local estimated values before the weighted fusion. Note that $\hat{x}_{k|k-1}^{i,L} \triangleq E(\hat{x}_{k|k-1}^{i,L})$ is a roughly estimated value according to the statistical characteristic of random noise variables derived from $\hat{x}_{k-1|k-1}^{i,L}$.
240

Using the reorganized measurement space $\mathcal{L} \{y_k^1, \dots, y_k^n, y_{k-1}^1, \dots, y_{k-1}^n\}$ and the weighted fusion reorganized innovation sequence $\{\tilde{\varepsilon}_k, \tilde{\varepsilon}_{k-1}\}$, the state estimate $\hat{x}_{k|k}^i$ is generated at time instant k :

$$\hat{x}_{k|k}^i = \hat{x}_{k|k-1}^i + K_k^i \tilde{\varepsilon}_k = A_{k-1} \hat{x}_{k-1|k-1}^i + K_k^i \Theta_k^T \tilde{Y}_k, \quad (17)$$

where $\hat{x}_{k|k}^i$ is obtained by the weighted fusion reorganized innovation, and K_k^i denotes the filtering gain matrix.

Taking expectation of error estimate as $E(\tilde{x}_{k|k}^i) = E(x_k - \hat{x}_{k|k}^i)$, since the characteristic of zero-mean white noise is $E(\tilde{\varepsilon}_k) = E\left(\sum_{i=1}^n \theta_k^i \varepsilon_k^i\right) = 0$, the
245 aforementioned result can be obtained $E(\tilde{x}_{k|k}^i) = 0$. Therefore, the estimated value of $\hat{x}_{k|k}^i$ by the weighted fusion reorganized innovation has the unbiased characteristic.

3.2.1. Kalman-type filter parameter

The considered distributed uncertain system with network-induced transmission time-delay is composed of the stochastic uncertainty and cross-correlated
250

noise signals, in order to minimize the filtering error covariance by one-step prediction compensation scheme, an appropriate filter parameter K_k^i of the i^{th} subsystem at time instant k can be found.

As mentioned before, $\hat{x}_{k|k-1}^{i,L}$ expresses the one-step prediction before being fused, and state x_{k-1} denotes an estimated value obtained from y_k , which is

$$x_k = \Psi_{k-1}x_{k-1} + B_{k-1}\omega_{k-1}, \quad (18)$$

where $\Psi_{k-1} \triangleq A_{k-1} + C_{k-1}\xi_{k-1}$. From (15), (16) and (18), a local robust Kalman-type recursive filter before the weighted fusion is designed by

$$\begin{aligned} \hat{x}_{k|k-1}^{i,L} &= A_{k-1} \hat{x}_{k-1|k-1}^{i,L}, \\ \hat{x}_{k|k}^{i,L} &= \Phi_k^i A_{k-1} \hat{x}_{k-1|k-1}^{i,L} + K_k^i y_k^i, \end{aligned} \quad (19)$$

in which $\Phi_k^i = I - K_k^i H_k^i$. Denoting the filtering error $\tilde{x}_{k-1|k-1}^{i,L} = x_{k-1} - \hat{x}_{k-1|k-1}^{i,L}$ is correlated with the process noise ω_{k-1} ; thus, the local one-step prediction error covariance $P_{k|k-1}^{i,L}$ before the weighted fusion can be calculated as follows:

$$\begin{aligned} P_{k|k-1}^{i,L} &= E \left(\tilde{x}_{k|k-1}^{i,L} \left(\tilde{x}_{k|k-1}^{i,L} \right)^T \right) \\ &= A_{k-1} P_{k-1|k-1}^{i,L} A_{k-1}^T + B_{k-1} (W_{k-1}^i)^T A_{k-1}^T + C_{k-1} X_{k-1} C_{k-1}^T + \\ &\quad A_{k-1} W_{k-1}^i B_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T, \end{aligned} \quad (20)$$

here $P_{k-1|k-1}^{i,L}$ expresses the local filtering error covariance at time instant $k-1$.

Moreover, the notations are defined in the following forms: $X_{k-1} \triangleq E(x_{k-1} x_{k-1}^T)$ and $W_{k-1}^i \triangleq E(\tilde{x}_{k-1|k-1}^{i,L} \omega_{k-1}^T)$.

The expectation X_{k-1} can be evolved from state x_{k-1} given in (18), and it is derived from:

$$\begin{aligned} X_k &= E(x_k x_k^T) \\ &= A_{k-1} X_{k-1} A_{k-1}^T + C_{k-1} X_{k-1} C_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T + \\ &\quad A_{k-1} B_{k-2} Q_{k-2,k-1} B_{k-1}^T + B_{k-1} Q_{k-1,k-2} B_{k-2}^T A_{k-1}^T. \end{aligned} \quad (21)$$

Note that the initial value of X_0 is determined by $X_0 \triangleq \mu_0 \mu_0^T + P_0$ [13].

The expectation W_{k-1}^i involves the cross-correlation between process noise ω_k and measurement noise v_k^i , which is represented as

$$W_{k-1}^i = B_{k-2}Q_{k-2,k-1} - K_{k-1}^i \left(H_{k-1}^i B_{k-2}Q_{k-2,k-1} + (S_{k-1}^i)^T \right). \quad (22)$$

Similar to the one-step prediction error covariance, the filtering error covariance $P_{k|k}^{i,L}$ is calculated by

$$\begin{aligned} P_{k|k}^{i,L} &= E \left(\tilde{x}_{k|k}^{i,L} (\tilde{x}_{k|k}^{i,L})^T \right) \\ &= \Phi_k^i P_{k|k-1}^{i,L} (\Phi_k^i)^T + K_k^i R_k^i (K_k^i)^T - \\ &\quad \Phi_k^i V_{k-1}^i (K_k^i)^T - K_k^i (V_{k-1}^i)^T (\Phi_k^i)^T, \end{aligned} \quad (23)$$

in which, the expectation $V_{k-1}^i \triangleq E \left(\tilde{x}_{k|k-1}^{i,L} (v_k^i)^T \right)$ can be obtained by

$$V_{k-1}^i = B_{k-1}S_{k-1,k}^i - A_{k-1}K_{k-1}^i R_{k-1,k}^i. \quad (24)$$

To design a local optimal filter, selecting an appropriate K_k^i is essential to minimize the filtering error covariance $P_{k|k}^{i,L}$ given in (23). And then, K_k^i is obtained by $\partial \left(Tr \left(P_{k|k}^{i,L} \right) \right) / \partial K_k^i = 0$, where function $Tr(P)$ denotes the trace operator of matrix P :

$$K_k^i = \Xi_{k-1}^i \left(H_k^i P_{k|k-1}^{i,L} (H_k^i)^T + R_k^i + \tilde{\Xi}_{k-1}^i \right)^{-1}, \quad (25)$$

where $\Xi_{k-1}^i = P_{k|k-1}^{i,L} (H_k^i)^T + V_{k-1}^i$ and $\tilde{\Xi}_{k-1}^i = H_k^i V_{k-1}^i + (V_{k-1}^i)^T (H_k^i)^T$.

Substituting (25) into (23), $P_{k|k}^{i,L}$ is then rewritten as

$$P_{k|k}^{i,L} = P_{k|k-1}^{i,L} - K_k^i (\Xi_{k-1}^i)^T. \quad (26)$$

Finally, with the aid of the above results, Σ_k given in (13) can be derived from (19)-(26):

$$\begin{aligned} \Sigma_k^{i,j} &= E \left(\varepsilon_k^i (\varepsilon_k^j)^T \right) = H_k^i P_{k|k-1}^{i,j,L} (H_k^j)^T + H_k^i V_{k-1}^{j,i} + (H_k^j V_{k-1}^{i,i})^T + R_k^{i,j}, \\ &\quad i, j = 1, \dots, n \end{aligned} \quad (27)$$

where Σ_k is an $nm \times nm$ -dimensional matrix and

$$V_{k-1}^{i,j} \triangleq E \left(\tilde{x}_{k|k-1}^{i,L} (v_k^j)^T \right) = B_{k-1}S_{k-1,k}^j - A_{k-1}K_{k-1}^i R_{k-1,k}^{i,j}. \quad (28)$$

3.2.2. Fusion estimation for filtering error cross-covariance

260 As mentioned the cross-correlation for noises, in order to improve the state estimation accuracy, the information fusion is necessary to probe the optimal distributed estimation. In this paper, the weighted fusion criterion is applied in the filtering error cross-covariance, which needs to calculate the filtering error cross-covariance matrix between any two subsystems for achieving information
265 exchange.

Theorem 1. *For the distributed linear discrete time-varying system (1) and (2), based on the filtering estimation by one-step compensation of $\hat{x}_{k-1|k-1}^i$ in (17) and filtering gain K_k^i in (25), the filtering error cross-covariance matrix $P_{k|k}^{i,j}$ and the prediction error cross-covariance matrix $P_{k|k-1}^{i,j}$ between the i^{th} and j^{th} subsystems at time instant k have the following expressions:*

$$\begin{aligned} \hat{x}_{k|k-1}^i &= A_{k-1} \hat{x}_{k-1|k-1}^i, \\ \hat{x}_{k|k}^i &= A_{k-1} \hat{x}_{k-1|k-1}^i + K_k^i \tilde{\varepsilon}_k, \\ K_k^i &= \Xi_{k-1}^i \left(H_k^i P_{k|k-1}^i (H_k^i)^T + R_k^i + \tilde{\Xi}_{k-1}^i \right)^{-1}, \\ P_{k|k-1}^{i,j} &= A_{k-1} P_{k-1|k-1}^{i,j} A_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T + \\ &\quad C_{k-1} X_{k-1} C_{k-1}^T + B_{k-1} \left(G_{k-1}^j \right)^T A_{k-1}^T + A_{k-1} G_{k-1}^i B_{k-1}^T, \\ P_{k|k}^{i,j} &= P_{k|k-1}^{i,j} + K_k^i P_k \left(K_k^j \right)^T - A_{k-1} M_{k-1}^i \left(K_k^j \right)^T - \\ &\quad K_k^i \left(M_{k-1}^j \right)^T A_{k-1}^T - B_{k-1} L_{k-1}^T \left(K_k^j \right)^T - K_k^i L_{k-1} B_{k-1}^T. \end{aligned} \quad (29)$$

where

$$\begin{aligned} P_k &= E \left(\tilde{\varepsilon}_k \tilde{\varepsilon}_k^T \right), \\ G_{k-1}^i &= B_{k-2} Q_{k-2,k-1} - K_{k-1}^i \sum_{p=1}^n \theta_{k-1}^p \left(H_{k-1}^p B_{k-2} Q_{k-2,k-1} + (S_{k-1}^p)^T \right), \\ X_k &= A_{k-1} X_{k-1} A_{k-1}^T + C_{k-1} X_{k-1} C_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T + \\ &\quad A_{k-1} B_{k-2} Q_{k-2,k-1} B_{k-1}^T + B_{k-1} Q_{k-1,k-2} B_{k-2}^T A_{k-1}^T, \\ M_{k-1}^i &= \sum_{p=1}^n \left(P_{k-1|k-1}^{i,p} (H_k^p A_{k-1})^T + G_{k-1}^i (H_k^p B_{k-1})^T + U_{k-1}^{i,p} \right) \theta_k^p, \\ L_{k-1} &= \sum_{p=1}^n \theta_k^p \left(H_k^p A_{k-1} G_{k-1}^p + H_k^p B_{k-1} Q_{k-1} + (S_{k-1,k}^p)^T \right), \\ U_{k-1}^{i,p} &= B_{k-1} S_{k-1,k}^p - K_{k-1}^i \sum_{p=1}^n \theta_{k-1}^q R_{k-1,k}^{q,p}, \quad p = 1, 2, \dots, n. \end{aligned} \quad (30)$$

Proof. The estimated values based on one-step prediction are evolved as $\hat{x}_{k|k}^i$ and $\hat{x}_{k|k}^j$ from the i^{th} and j^{th} subsystems, respectively. Without loss of generality, the reorganized robust Kalman-type recursive filters are calculated as:

$$\begin{aligned}\hat{x}_{k|k-1}^i &= A_{k-1}\hat{x}_{k-1|k-1}^i, \\ \hat{x}_{k|k}^i &= A_{k-1}\hat{x}_{k-1|k-1}^i + K_k^i\tilde{\varepsilon}_k,\end{aligned}\tag{31}$$

and

$$\begin{aligned}\hat{x}_{k|k-1}^j &= A_{k-1}\hat{x}_{k-1|k-1}^j, \\ \hat{x}_{k|k}^j &= A_{k-1}\hat{x}_{k-1|k-1}^j + K_k^j\tilde{\varepsilon}_k.\end{aligned}\tag{32}$$

Similar to (20) and (23), the corresponding one-step prediction and filtering error cross-covariance matrices between the i^{th} and j^{th} subsystems are formulated, respectively, in the following forms at time instant k :

$$\begin{aligned}P_{k|k-1}^{i,j} &= E\left(\tilde{x}_{k|k-1}^i\left(\tilde{x}_{k|k-1}^j\right)^T\right) \\ &= A_{k-1}P_{k-1|k-1}^{i,j}A_{k-1}^T + B_{k-1}Q_{k-1}B_{k-1}^T + C_{k-1}X_{k-1}C_{k-1}^T + \\ &\quad B_{k-1}\left(G_{k-1}^j\right)^TA_{k-1}^T + A_{k-1}G_{k-1}^iB_{k-1}^T,\end{aligned}\tag{33}$$

and

$$\begin{aligned}P_{k|k}^{i,j} &= E\left(\tilde{x}_{k|k}^i\left(\tilde{x}_{k|k}^j\right)^T\right) \\ &= P_{k|k-1}^{i,j} + K_k^iP_k\left(K_k^j\right)^T - A_{k-1}M_{k-1}^i\left(K_k^j\right)^T - \\ &\quad K_k^i\left(M_{k-1}^j\right)^TA_{k-1}^T - B_{k-1}L_{k-1}^T\left(K_k^j\right)^T - K_k^iL_{k-1}B_{k-1}^T.\end{aligned}\tag{34}$$

Define $G_{k-1}^i \triangleq E\left(\tilde{x}_{k-1|k-1}^i\omega_{k-1}^T\right)$, $M_{k-1}^i \triangleq E\left(\tilde{x}_{k-1|k-1}^i\tilde{\varepsilon}_k^T\right)$, $X_{k-1} \triangleq E\left(x_{k-1}x_{k-1}^T\right)$, $P_k \triangleq E\left(\tilde{\varepsilon}_k\tilde{\varepsilon}_k^T\right)$ and $L_{k-1} \triangleq E\left(\tilde{\varepsilon}_k\omega_{k-1}^T\right)$. From Lemma 1, P_k is obtained by (13).

Taking Remark 1 and cross-correlated noises into account that the expectation

G_{k-1}^i , M_{k-1}^i , X_{k-1} and L_{k-1} can be derived from the following forms:

$$G_{k-1}^i = B_{k-2}Q_{k-2,k-1} - K_{k-1}^i\sum_{p=1}^n\theta_{k-1}^p\left(H_{k-1}^pB_{k-2}Q_{k-2,k-1} + (S_{k-1}^p)^T\right),\tag{35}$$

where $X_{k-1} = E\left(x_{k-1}x_{k-1}^T\right)$ and X_k is equivalent to (21).

And then, the expectation M_{k-1}^i represents the cross-correlated matrix between one-step estimation error and fused innovation sequence. Also, the expectation L_{k-1} is the cross-correlated matrix between the fused innovation sequence

and process noise. From (11), they can be written as

$$M_{k-1}^i = \sum_{p=1}^n \left(P_{k-1|k-1}^{i,p} (H_k^p A_{k-1})^T + G_{k-1}^i (H_k^p B_{k-1})^T + U_{k-1}^{i,p} \right) \theta_k^p, \quad (36)$$

and

$$L_{k-1} = \sum_{p=1}^n \theta_k^p \left(H_k^p A_{k-1} G_{k-1}^p + H_k^p B_{k-1} Q_{k-1} + \left(S_{k-1,k}^p \right)^T \right). \quad (37)$$

Note that

$$\begin{aligned} U_{k-1}^{i,p} &= E \left(\tilde{x}_{k-1|k-1}^i (v_k^p)^T \right) \\ &= B_{k-1} S_{k-1,k}^p - K_{k-1}^i \sum_{p=1}^n \theta_{k-1}^q R_{k-1,k}^{q,p}, \quad p = 1, 2, \dots, n. \end{aligned} \quad (38)$$

Substituting (21), (35)-(37) into (33) and (34), respectively, yields the fused Kalman-type filter given in (29). The solutions of Theorem 1 are proved. \square

Remark 4. Based on the traditional Kalman filtering, the local filter of the distributed networked system before the weighted fusion is (15) and (16), and filtering gain K_k^i is obtained. And then, to achieve the information exchanges between neighboring sensor nodes, Eqs. (31) and (32) are the reorganized robust Kalman-type recursive filters, which are derived from reorganized innovation sequence $\tilde{\varepsilon}_k$. In addition, the error cross-covariance matrices are used to obtain the information collaboration from (33) and (34) depending on the weighted matrix Θ_k .

With the aid of the optimal weighted fusion filtering method in the linear minimum variance [14, 42], the following Theorem 2 can be used to verify the optimal weighted matrix to minimize the trace of the fusion filtering error cross-covariance.

Theorem 2. Let $\hat{x}_{k|k}^i$, $i = 1, \dots, n$ represent the unbiased estimation of an r -dimensional stochastic vector x . The optimal distributed fusion estimator is denoted as $\hat{x}_{k|k}$, which is also an r -dimensional vector. Denote

$$J = (I x_k - \hat{x}_{k|k})^T \Sigma_{k|k}^{-1} (I x_k - \hat{x}_{k|k}), \quad (39)$$

where $I = \underbrace{[I_r, \dots, I_r]^T}_n$ is an $rn \times r$ matrix, and

$$\Sigma_{k|k} = \begin{bmatrix} P_{k|k}^{1,1} & \dots & P_{k|k}^{1,n} \\ \vdots & & \vdots \\ P_{k|k}^{n,1} & \dots & P_{k|k}^{n,n} \end{bmatrix} = \left(P_{k|k}^{i,j} \right)_{n \times n} \quad (40)$$

is a symmetrical positive definite matrix. The corresponding covariance of the optimal information fusion estimator $P_{k|k}$ satisfies $P_{k|k} \leq P_{k|k}^i$, $i = 1, 2, \dots, n$ if $i = j$, otherwise, $P_{k|k} \leq P_{k|k}^{i,j}$.

This theorem follows from the weighted fusion criterion, which is proved in Appendix A.

The distributed robust Kalman-type filtering fusion structure can be illustrated in Fig.3. Note that y_k^i , $i = 1, \dots, n$ is the measurement output of the i^{th} sensor, $\hat{x}_{k|k}^{i,L}$ denotes the local estimated values before the fusion, and $P_{k|k-1}^{i,L}$ is the corresponding filtering error covariance; meanwhile, $\hat{x}_{k|k}^i$ and $P_{k|k}^{i,j}$ are the fused estimation and filtering error cross-covariance by the weighted fusion reorganized innovation sequence, respectively. Moreover, $\hat{x}_{k|k}$ and $P_{k|k}$ represent the weighted fusion state estimation and error cross-covariance in the fusion center.

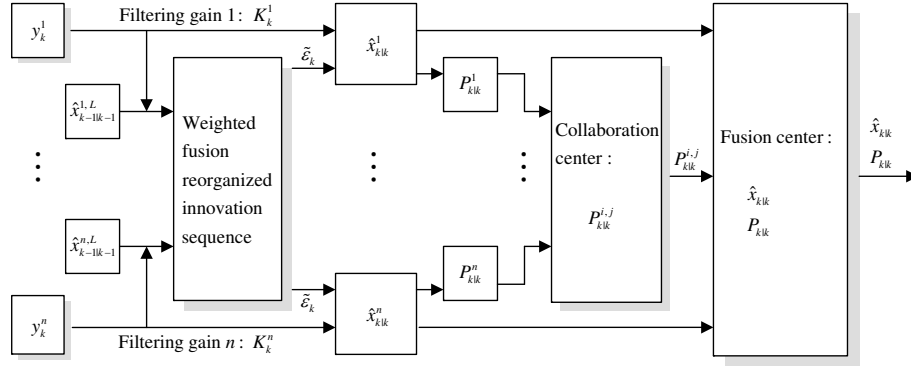


Figure 3: Distributed weighted fusion estimation based on robust Kalman one-step compensation.

Based on Theorem 1 and Theorem 2, the computation procedure for the distributed fusion estimation is shown in Algorithm I.

Algorithm 1 Distributed fusion estimation computation procedure

Input: The initial state $x_{0|0}$, mean μ_0 , variance $P_{0|0}$, and sample time t

Output: State estimation $\hat{x}_{k|k}$ and estimation error cross-covariance $P_{k|k}$

for $k = 1 \rightarrow t$ **do**

for $i = 1 \rightarrow n$ **do**

$\hat{x}_{k|k-1}^{i,L}$ in (15); // one-step prediction estimator before the fusion

$\hat{x}_{k|k}^{i,L}$ in (16); // filtering estimator before the fusion

K_k^i in (25); // filtering gain

end for

$\tilde{\varepsilon}_k$ given in (11); // weighted fusion reorganized innovation sequence

for $i = 1 \rightarrow n$ **do**

$\hat{x}_{k|k-1}^i$ and $\hat{x}_{k|k}^i$ in (31); // state estimator of the i^{th} subsystem

$P_{k|k-1}^{i,j}$ in (33) and $P_{k|k}^{i,j}$ in (34);

 // prediction and filtering error cross-covariance matrices

end for

$\hat{x}_{k|k} = \sum_{i=1}^n \Lambda_k^i \hat{x}_{k|k}^i$ in (A.2); // weighted fusion state estimation

$P_{k|k} = \left(I^T \Sigma_{k|k}^{-1} I \right)^{-1}$ in (A.4);

 // weighted fusion estimation error cross-covariance

end for

Remark 5. The distributed fusion estimation based on robust Kalman-type filtering presents a two-level weighted fusion strategy with the reorganized innovation sequence and estimation error cross-covariance. Depending on the above proofs, the weighted fusion criteria have higher estimation accuracy than fusion filter by the linear minimum variance matrices [14, 42]. The cross-correlated measurement noises over sensor networks are introduced at the current and previous time instants, and the transmission time-delays are simultaneously considered in a class of stochastic uncertain systems. Note that the limitations

of the proposed approach are the assumptions of the noise variances with exactly known and the measurement with constant transmission time-delay at each sample time. However, when parallel computation is available, the distributed robust Kalman-type filter method for fusion estimation could be flexible and robust.

3.2.3. Multi-step time-delays

The measurement transmitted from plant to estimator via networked systems due to the limited communication capability, random delays may exist at different sample time. When the time-delay $\tau_k \geq 2$ at time instant k , the distributed system from (1) and (2) is able to transform into Kalman multi-step predictors given by the reorganized measurement sequence and innovation sequence.

Considering the reorganized linear measurement space $\mathcal{L}\{y_k^i, \dots, y_{k-\tau_k}^i\}$, the unbiased estimation $\hat{x}_{k|k}^i$ will be compensated by multi-step prediction of $\hat{x}_{k-1|k-1}^i$ at time instant k . The multi-step Kalman-type predictors are necessary to be substituted into the one-step robust Kalman predictor, and it is defined as

$$\hat{x}_{k|k}^i = \prod_{p=1}^{\tau_k} A_{k-p} \hat{x}_{k-\tau_k|k-\tau_k}^i + K_k^i \varepsilon_k^i = \Phi_k^i \prod_{p=1}^{\tau_k} A_{k-p} \hat{x}_{k-\tau_k|k-\tau_k}^i + K_k^i y_k^i. \quad (41)$$

The state x_k to be estimated is calculated by

$$x_k = \prod_{p=1}^{\tau_k} \Psi_{k-p} x_{k-\tau_k} + \sum_{q=1}^{\tau_k-1} \left(\prod_{p=1}^{\tau_k-q} \Psi_{k-p} \right) B_{k-\tau_k+q-1} \omega_{k-\tau_k} + B_{k-1} \omega_{k-\tau_k}, \quad (42)$$

where $\Psi_k = A_k + \Delta A_k$.

Subtracting (41) from (42), the multi-step prediction error is denoted as

$$\tilde{x}_{k|k}^i = x_k - \hat{x}_{k|k}^i = \Phi_k^i \left(x_k - \prod_{p=1}^{\tau_k} A_{k-p} \hat{x}_{k-\tau_k|k-\tau_k}^i \right) - K_k^i v_k^i, \quad (43)$$

where $\Phi_k^i = I - K_k^i H_k^i$. According to the estimation error $\tilde{x}_{k-\tau_k|k-\tau_k}^i$, the conservative local steady-state error $\tilde{x}_{k|k}^i$ is derived by multi-step prediction error covariances as $P_{k|k}^i = E \left(\tilde{x}_{k|k}^i \left(\tilde{x}_{k|k}^i \right)^T \right)$, which is correlated with $P_{k-\tau_k|k-\tau_k}^i$.

In order to reduce the computational burden and simplify the augmented state, the time-varying multi-step transmission delays could be transformed into the one-step time-delay by the compensation strategy.

4. Numerical simulation

In this section, the effectiveness of the proposed distributed weighted fusion estimation for robust Kalman filtering approach is demonstrated by a numerical example.

The following radar tracking system with four sensors is described in [13, 14, 27, 42]. The distributed networked system with transmission time-delay, stochastic uncertainties and cross-correlated noises is given by:

$$x_{k+1} = \left(\begin{bmatrix} 0.96 & T & T^2/2 \\ 0 & 0.96 & T \\ 0 & 0 & 0.96 \end{bmatrix} + \Delta A_k \right) x_k + \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix} \omega_k, \quad (44)$$

$k = 1, 2, \dots$

$$y_k^i = H_k^i x_{k-\tau_k} + v_k^i, \quad i = 1, 2, 3, 4, \quad (45)$$

$$\Delta A_k = C_k \xi_k, \quad (46)$$

$$\omega_k = \eta_k + \eta_{k-1}, \quad (47)$$

$$v_k^i = \beta_i \omega_{k-1} + \beta_i \eta_k, \quad i = 1, 2, 3, 4, \quad (48)$$

where symbol T denotes the sampling period chosen as $0.1s$, and the state $x_k = (s_k \ \dot{s}_k \ \ddot{s}_k)^T$ is composed of the position, velocity and acceleration, respectively, of the target at time instant kT . $\xi_k \in \Re$ represents the state-dependent noise, which is white noise with zero-mean and variance $\sigma_\xi^2 = 1$.

$\eta_k \in \Re$ is also zero-mean white noise with variance $\sigma_\eta^2 = 0.09$, and is independent of ξ_k . Set $C = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$ be the state transition matrix from system internal uncertainties. y_k^i ($i = 1, 2, 3, 4$) are the measurements of the four sensors with time-delay $\tau_k = 1$, which measure the position, velocity and acceleration, respectively. The measurement matrices are set as $H_1 = [1 \ 0.6 \ 0.8]$,
 $H_2 = [0.8 \ 1 \ 0.5]$, $H_3 = [0.8 \ 0.3 \ 1]$ and $H_4 = [0.6 \ 0.5 \ 1]$. In the considered system, the measurement noise v_k^i of the i^{th} sensor is cross-correlated with the process noise ω_k , and is also cross-correlated with v_k^j of the j^{th} sensor for one sample time only. The variable β_i in (48) determines the strength of the correlation and is set as $\beta_1 = 1$, $\beta_2 = 0.8$, $\beta_3 = 2$ and $\beta_4 = 5$.

Without loss of generality, the process noise ω_k from (47) has the variance Q_k , and the covariance $R_k = (R_k^i)_{n \times n}$ of measurement noise between v_k^i and v_k^j given in (48) is obtained by

$$R_k = \begin{bmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & \beta_1\beta_4 \\ \beta_2\beta_1 & \beta_2^2 & \beta_2\beta_3 & \beta_2\beta_4 \\ \beta_3\beta_1 & \beta_3\beta_2 & \beta_3^2 & \beta_3\beta_4 \\ \beta_4\beta_1 & \beta_4\beta_2 & \beta_4\beta_3 & \beta_4^2 \end{bmatrix} (Q_k + \sigma_{\eta_k}^2) . \quad (49)$$

Similarly, the cross-covariance between the process noise and measurement noise is

$$S_k = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4]^T (\sigma_{\eta_{k-1}}^2 + \sigma_{\eta_k}^2) . \quad (50)$$

For simulation purpose, the initial values are set as $\hat{x}_{0|0} = \mu_0 = E(x_0) = [1 \ 1 \ 1]^T$ and $P_{0|0} = 0.01I_3$. The proposed distributed robust Kalman filtering (DRKF) is simulated and compared with the optimal sequential fusion (OSF) [14] and weighted robust Kalman filter (WRKF) [13] algorithms. 300 time sampling points are taken, and the results are obtained based on 300 Monte Carlo simulations.

Fig.4 shows cross-correlation between process noise and measurement noise, where the cross-correlation is described from (3), (47) and (48). In order to

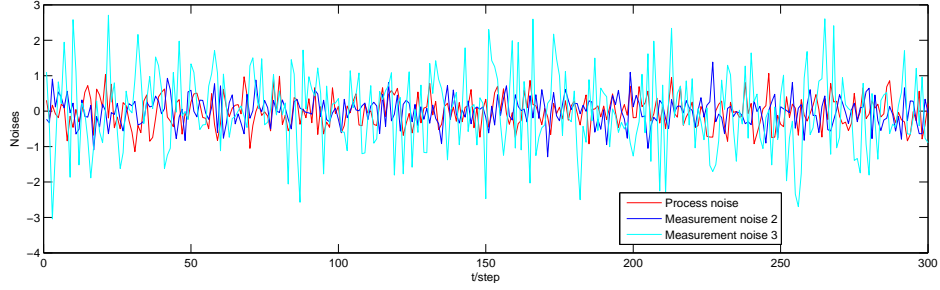


Figure 4: Cross-correlation for process noise and measurement noise.

clearly explain the transmission time-delay and cross-correlation, the process
 350 and measurement noises on the second sensor and the third sensor are also
 plotted.

The filter results of the acceleration state $\ddot{s}(k)$ are shown in Fig.5.

To demonstrate the proposed weighted fusion reorganized innovation and the
 error covariance schemes improving the estimation accuracy, Fig.5 (a) illustrates
 355 that the proposed distributed robust Kalman-type filter approach has the best
 target tacking performance with transmission time-delay and cross-correlated
 noises. Meanwhile, Figs.5 (b)-(d) show the estimated state from each sensor,
 which compare with the state-dependent noise and noise-free cases. Note that
 the state-dependent noise-free generates the change of the state trajectory, thus,
 360 the estimated value with state-dependent noise-free is unable to follow well the
 system dynamics. Due to the existence of transmission time-delay, the state
 to be estimated contains peak and trough; meanwhile, the cross-correlation for
 noises makes the tracking path curve not smooth. When the strength of cross-
 correlation for noises is a small value, the estimation for target tracking is mainly
 365 dependent with the distributed robust Kalman filter method.

In order to further verify the effectiveness of the proposed distributed weight-
 ed fusion estimation method, the comparative analysis of estimation error co-
 variances results are displayed in Fig.6. Fig.6 (a) demonstrates the comparison

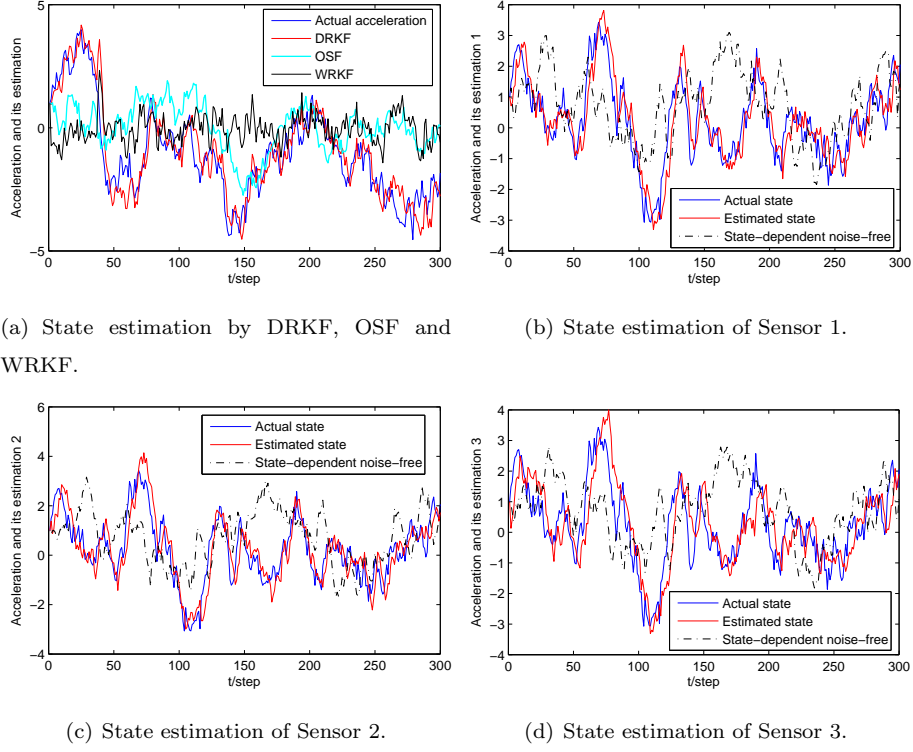
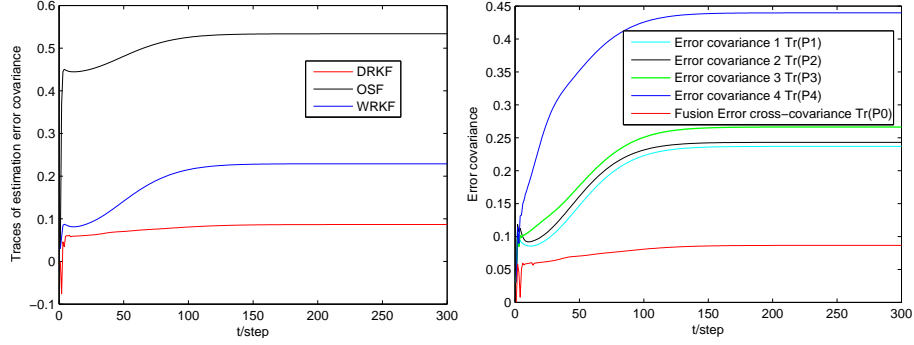


Figure 5: Comparison of acceleration state and its estimation.

between WRKF, OSF and proposed DRKF methods. And Fig.6 (b) illustrates
 370 the error covariances between each filter and the fusion estimation for DRKF.

The proposed estimation method makes use of the weighted fusion reorga-
 nized innovation sequence, which fuses the transmission disturbances from each
 sensor to suppress the effect of noises on measurements. As shown in Fig.5 (a)
 and Fig.6 (a), the proposed weighted fusion method is able to quickly converge
 375 to a stable state, so that the jump phenomenon is weakened during the early
 sample time intervals. Furthermore, the optimal information weighted fusion
 estimator is used to obtain the robustness and optimal steady-state value.

The performance indicator of robustness and actual accuracies are verified
 by the mean square error (MSE) value [16] depending on the actual state values



(a) Estimation error covariances from OSF, (b) Estimation error covariances of DRKF WRKF and DRKF from each sensor and fusion center.

Figure 6: Comparison of filtering error covariance.

and estimated values, which is calculated as

$$MSE = \frac{1}{q} \sum_{i=1}^q \left((x_k - \hat{x}_{k|k})^T (x_k - \hat{x}_{k|k}) \right). \quad (51)$$

where q denotes the sampling points. When $k \rightarrow \infty$ and $q \rightarrow \infty$ given in the ergodicity, MSE tends to the trace of the state estimation error covariance, that is $MSE \rightarrow Tr(P)$. The MSE values obtained from the state of acceleration are shown in Table 1, and the trace values are listed in Table 2. Note that MSE and trace values are derived from each sensor and the fusion center.

Table 1 Comparison of the filter mean square errors for the state of acceleration

Method	OSF	WRKF	DRKFFE
Sensor 1	2.1420	1.6108	0.2385
Sensor 2	2.1413	1.6050	0.2464
Sensor 3	2.1397	1.6673	0.3229
Sensor 4	2.1502	1.5650	0.4550
Fused MSE	1.0153	0.3313	0.2056

Table 2 Comparison of the trace for estimation error covariance

Method	OSF	WRKF	DRKFFE
Tr(P1)	0.1553	0.1515	0.1476
Tr(P2)	0.1550	0.1544	0.1479
Tr(P3)	0.2010	0.1617	0.1596
Tr(P4)	0.3257	0.2111	0.2105
Tr(P0)	0.1534	0.1084	0.0943
Trace of estimation error covariance	0.2055	0.1626	0.0941

From Table 1 and Table 2, it can be obtained that the local and fused estimation satisfy $MSE_{DRKF} < MSE_{WRKF} < MSE_{OSF}$ and $Tr(P)_{DRKF} <$
385 $Tr(P)_{WRKF} < Tr(P)_{OSF}$. Due to the two level weighted fusion is again optimal in the minimum covariance sense, the estimation accuracy is higher than that of each local robust Kalman-type filter, and is also higher than that of WRKF and OSF methods.

According to the simulation results, as well as the MSE and the trace of
390 estimation error covariance criteria, it is summarized that the proposed distributed robust Kalman-type filter for fusion estimation provides the optimal performances to rapidly converge to a steady-state, alleviate the uncertain disturbances and yields a higher measurement precision than each local estimation.

5. Conclusion

With the aid of robust Kalman filtering, the distributed perception and cen-
395 tralized fusion estimation strategies are investigated. To solve the optimal state estimation issue, the distributed stochastic uncertain systems for sensor networks with transmission time-delay and cross-correlated noises introduce two level weighted fusion schemes, which include reorganized innovation sequence
400 and filtering error cross-covariance. Based on the distributed information fusion and innovation sequence reorganization criteria, the proposed approach is used to reduce communication burden of the large-scale networked systems, suppress the noises disturbances, lower computational complexity, and further improve the estimation accuracy. As a theoretical result, the proposed method
405 possesses flexibility, robustness and scalability. Numerical simulations illustrate that the distributed weighted fusion estimation has the capability of tracking the dynamic state of the system at the current time instant.

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415 Appendix A. Proof of Theorem 2.

For each sampling period $k = 1, 2, \dots$, set x_k be a convex function. The objective function J in (39) is a quadratic equation based on state x_k . Therefore, introduce the first derivative of x_k to probe the minimizing objective function J . Set $\partial J / \partial x_k = 0$ to obtain

$$x_k = \left(I^T \Sigma_{k|k}^{-1} I \right)^{-1} I^T \Sigma_{k|k}^{-1} \hat{x}_{k|k}. \quad (\text{A.1})$$

Note that the covariance $\Sigma_{k|k}$ is the non-zero matrix owing to the external disturbances would give rise to non-zero filtering error. The involved optimal weighted matrix [15] Λ_k represents the weight of distributed estimation $\hat{x}_{k|k}^i$, which is denoted as

$$\hat{x}_{k|k} = \sum_{i=1}^n \Lambda_k^i \hat{x}_{k|k}^i, \quad (\text{A.2})$$

where $\Lambda_k = [\Lambda_k^1, \dots, \Lambda_k^n]^T$, and $\Lambda_k = \left(I^T \Sigma_{k|k}^{-1} I \right)^{-1} I^T \Sigma_{k|k}^{-1}$ given in (A.1) is an $r \times n$ matrix.

Furthermore, based on the unbiasedness, the estimation error is expressed as

$$\tilde{x}_{k|k} = \sum_{i=1}^n \Lambda_k^i \tilde{x}_{k|k}^i. \quad (\text{A.3})$$

The error covariance of the distributed estimation can be calculated as

$$P_{k|k} = \sum_{i=1}^n \Lambda_k^i P_{k|k}^{i,j} \left(\Lambda_k^j \right)^T = \left(I^T \Sigma_{k|k}^{-1} I \right)^{-1}. \quad (\text{A.4})$$

Adopting the Schwartz matrix inequality, the derivation is scaled by the following inequality:

$$\begin{aligned} P_{k|k} &= \left(I^T \Sigma_{k|k}^{-1} I \right)^{-1} = \tilde{\Sigma}_{k|k}^T \left(\left(\Sigma_{k|k}^{-1/2} I \right)^T \left(\Sigma_{k|k}^{-1/2} I \right) \right)^{-1} \tilde{\Sigma}_{k|k} \\ &\leq \left(\left(\Sigma_{k|k}^{1/2} I_i \right)^T \left(\Sigma_{k|k}^{1/2} I_i \right) \right) = P_{k|k}^i, \end{aligned} \quad (\text{A.5})$$

where $\tilde{\Sigma}_{k|k} = \left(\Sigma_{k|k}^{-1/2} I\right)^T \left(\Sigma_{k|k}^{1/2} I_i\right)$, and $I_i = \underbrace{[0, \dots, I_r, \dots, 0]^T}_n$ is an $r \times n$ matrix whose i^{th} element is I_r . Note that $\Lambda_k^i = I_r$ and $\Lambda_k^j = 0$ for $j = 1, 2, \dots, n, j \neq i$ in (A.1). Therefore, the fused error cross-covariance is able to satisfy $P_{k|k} = P_{k|k}^i$ if $i = j$, otherwise, $P_{k|k} \leq P_{k|k}^{i,j}$.

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